Euler characteristic: $\mathcal{X} = V - E + F$

$\mathcal{X}(\Sigma_{g,k}) = 2 - 2g - k$

$\mathcal{X}(N_{h,k}) = 2 - h - k$

P: $\mathbf{X} \to \mathbf{Y}$ is a **covering** (of X) if each $x \in \mathbf{X}$ has a trivially covered neighborhood **U**
i.e.

$$
\begin{array}{ccc}
P^{-1}(\mathbf{U}) & \cong & \mathbf{U} \times \mathbf{T} \\
 & {\scriptstyle P} \searrow \quad \swarrow {\scriptstyle \pi_1} & \\
 & \mathbf{U} &
\end{array}
$$

(T is a discrete space)

k-sheeted cover is one, in which $\#\mathbf{T} = k$

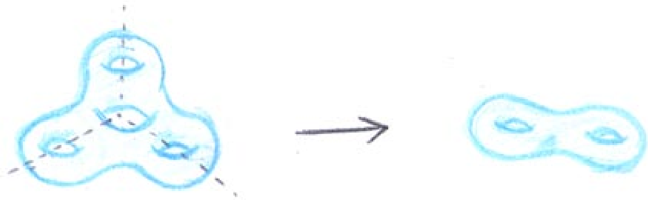
Any triangulation of **X** lifts to a triangulation of **Y**

$\mathcal{X}(\mathbf{Y}) = kV - kE + kF = k\,\mathcal{X}(\mathbf{X})$

Example: $\mathbb{S}^2 \to \mathbb{R}P^2$ is a 2-sheeted covering

$\mathcal{X}(\mathbb{R}P^2) = 3 - 6 + 4 = 1$

If a discrete group **G** acts on **Y** with no fixed points, then the quotient map $\mathbf{X} \to \mathbf{Y}/\mathbf{G}$ is a covering.



$\mathcal{X}(\Sigma_{4,0}) = -6$ $\qquad$ $\mathcal{X}(\Sigma_{2,0}) = -2$

works also with boundary

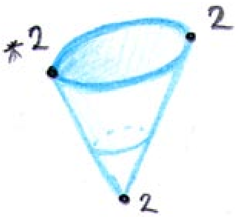
quotient by mirror

$\mathcal{X} = 2$ $\qquad$ $\mathcal{X} = 1$

$\mathcal{X} = -2$ $\qquad$ $\mathcal{X} = -1$

Orbifold euler chacteristic: $\chi_0(\mathbf{X}) = V_0 - E_0 + F_0$
but certain vertices and edges count less than 1
More precisley: choose a triangulation of **X** such that each point or corner point is a vertex and each mirror boundary curve is a union of edges.

In V_0 a vertex at a cone point(n) counts only $1/n$
An edge along a mirror line counts only $1/2$ in E_0
A vertex at a corner(∗n) counts $1/2n$



(2∗22)

$V = 3$
$E = 4$
$F = 2$

$\chi = 1$

$V_0 = 1/2 + 1/4 + 1/4 = 1$
$E_0 = 1 + 1 + 1/2 + 1/2 = 3$
$F_0 = F = 2$

$\chi_0 = 0$

Claim: If **G** is a discrete group of order k acting faithfully (non-trivial) on a surface **Y**, then
$\chi(\mathbf{X}) = \chi_0(\mathbf{X}) = k\,\chi_0(\mathbf{Y/G})$

Computing χ_0 from orbifold notation:

- Start with 2
- Substract 2 for each handle
- Substract 1 for each cross cap (x)
- Substract 1 for each mirror boundary (∗)
- Substract $1 - 1/n$ for each cone point (n)
- Substract $1 - 1/2n$ for each corner point (∗n)

Remark on mirror boundary

$\chi = V - E = n - n$

$\chi_0 = V_0 - E_0 = n/2 - n/2$

Example: $\chi_0(\mathbf{2{*}22}) = 2 - 1/2 - 1 - 1/4 - 1/4 = 0$

Which compact orbifolds have $\chi_0 = 0$?
(Think which features are you able to buy with the price-list above with 2$)

o	**2222**	**∗2222**	**4∗2**	**22∗**
xx	**236**	**∗236**	**2∗22**	**22x**
x∗	**244**	**∗244**		
∗∗	**333**	**∗333**	**3∗3**	

Exercise: Show that this list is really complete

Which compact orbifolds have $\mathcal{X}_0 > 0$?

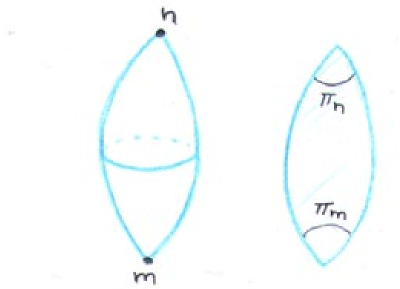
- $\mathcal{X}_0$ (**n∗**) = $^1/_n$ = $\mathcal{X}_0$ (**n×**) = $^2/_{|G|}$ ⇒ $|G| = 2n$
- $\mathcal{X}_0$ (**233**) = $2 - {}^1/_2 - {}^2/_3 - {}^2/_3 = {}^1/_6$ $|G| = 12$
- $\mathcal{X}_0$ (**234**) = $2 - {}^1/_2 - {}^2/_3 - {}^3/_4 = {}^1/_{12}$ $|G| = 24$
- $\mathcal{X}_0$ (**235**) = $^1/_{30}$ $|G| = 60$
- $\mathcal{X}_0$ (**∗233**) = $^1/_{12}$ $|G| = 24$
- $\mathcal{X}_0$ (**∗234**), $\mathcal{X}_0$ (**∗235**), $\mathcal{X}_0$ (**3∗2**)

- $\mathcal{X}_0$ (**∗nn**) = $2 - 1 - 2\,({}^1/_2 - {}^1/_{2n}) = {}^1/_n$ $|G| = 2n$
- $\mathcal{X}_0$ (**nn**) = $2 - 2\,(1 - {}^1/_n) = {}^2/_n$ $|G| = n$
- $\mathcal{X}_0$ (**2∗n**) = $2 - 1 - {}^1/_2 - ({}^1/_2 - {}^1/_{2n}) = {}^1/_{2n}$ $|G| = 4n$
- $\mathcal{X}_0$ (**∗22n**) = $^1/_{2n}$ $|G| = 4n$
- $\mathcal{X}_0$ (**22n**) = $^1/_n$ $|G| = 2n$

with any two cone points:

- $\mathcal{X}_0$ (**mn**) = $2 - (1 - {}^1/_n) - (1 - {}^1/_m) = {}^1/_n + {}^1/_m$
- $\mathcal{X}_0$ (**∗mn**) = $2 - 1 - {}^1/_2 (1 - {}^1/_n) - {}^1/_2 (1 - {}^1/_m) = {}^1/_{2n} + {}^1/_{2m}$

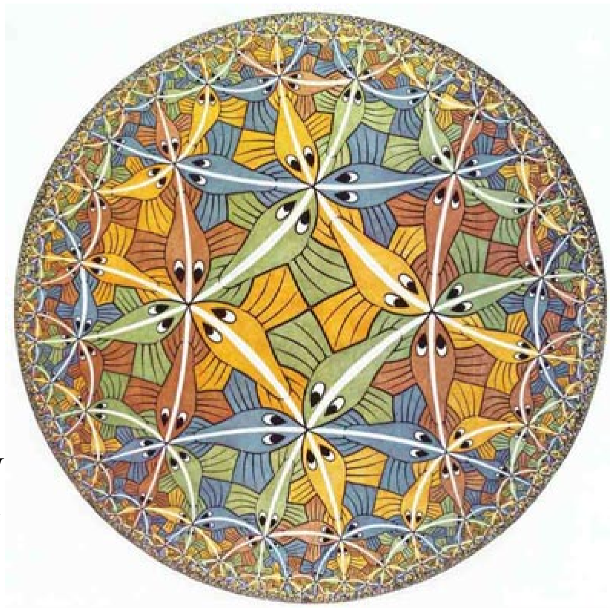
For $n > m \geqslant 1$: **mn** and **∗mn** are „bad“ manifolds



$\mathcal{X}_0 > 0$
But these are not quotients $\mathbb{S}^2/G$

Theorem: Every compact 2-orbifold **X** (except these bad manifolds) is a quotient of $\mathbb{S}^2$ (if $\mathcal{X}_0 > 0$), or $\mathbb{E}^2$ (if $\mathcal{X}_0 = 0$) or $\mathbb{H}^2$ (if $\mathcal{X}_0 < 0$) by some discrete group of isometries.

∗444 and **237** are „nice“ hyperbolic symetry groups



Circle Limits III
by M.C. Escher(with some help from H.S.M. Coxeter)