Mathematical Visualization WS 12/13 lecture notes

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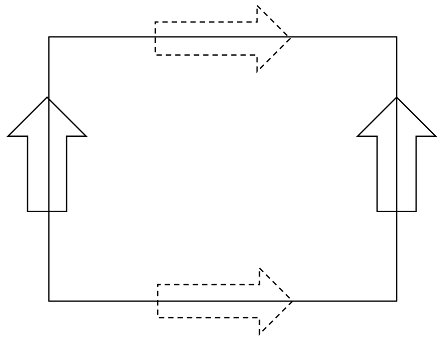
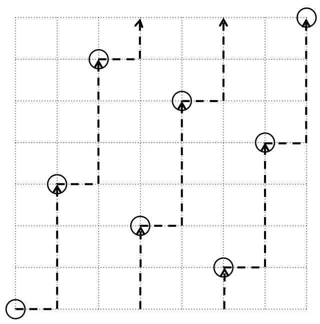
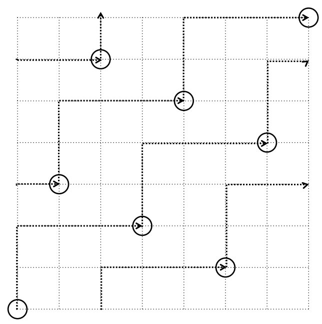
1 Lecture 29 Oct. 2012



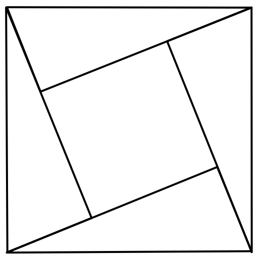
(p,q) - curve
(pt,qt)
k point spaced

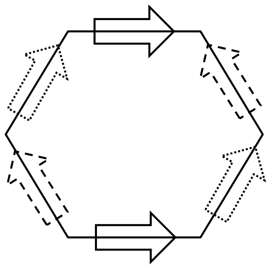
$$\left(\frac{pj}{k}, \frac{qj}{k}\right) \quad j = 1, \ldots, k$$



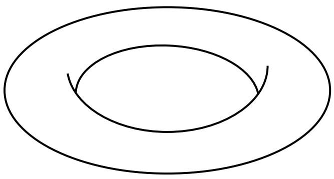


7 points on 2,3 curve Torus equal pointer [...]





$$\begin{pmatrix} (R + r\cos\theta)\cos\varphi \\ r\sin\theta \\ (R + r\cos\theta)\sin\varphi \end{pmatrix}$$

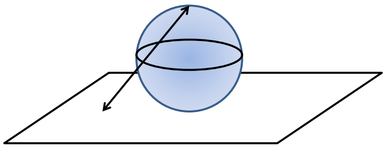


Toris in $\mathbb{S}^3$

$$\frac{\cos\theta, \sin\theta, \cos\varphi, sin\varphi}{\sqrt{2}}$$

$\cos\alpha\cos\theta, \cos\alpha\sin\theta, \sin\alpha\cos\varphi, \sin\alpha\sin\varphi$
$\alpha = \frac{\pi}{4} \Rightarrow$ square torus
$\alpha =$ other $\Rightarrow$ rectangular tori

to get a torus in $\mathbb{R}^3$ use stereografic geometrie



euclidean motion

$x \Rightarrow v + Mx, \quad v \in \mathbb{R}^n M \in O(n), \quad det(M) = \pm 1$
as a set, $\quad E_n = \mathbb{R}^n \times O(n)$
$(w, N)(v, M) = (wxNv, NM)$

$x \Rightarrow w + N(v + Mx) = (W + Nv) + (MN)x$
semidirect product

If $g, h \in G$ the conjugate of h by g is

$$[ghg^{-1}]$$

If G is abelien $gh = hg$ the conjugation is trivial $ghg^{-1} = h$) For fixed g the map $h \Rightarrow ghg^{-1}$ is an automorphism.

Conjugation by varying elements $g \in G$ gives a homomorphism

$$G \Rightarrow Aut(G)$$

G acts on its self

$$g \Rightarrow h \Rightarrow ghg^{-1}$$

The orbits of this actim are called *conjugacy classes*

$$\{ghg^{-1} : \quad g \in G\}$$

and should be thought of as geometrically similan motions.

$$G \Rightarrow Sym(x), \quad \tau_v \in E_n, \ |v| = 1$$

$$G \cdot x = g \cdot x : \quad g \in G$$

conjugacy class is all translations by unit vectors.
μ_P : reflection across hyperplane P
conjugacy class $(\mu_P) = \{$all reflections$\}$
p_r : rotations by $\frac{\pi}{2}$ around 0
conjugacy class $\{$rotations by $\frac{\pi}{2}$ around some $p\}$

Give $H \subset G$ and $g \in G$
$gHg^{-1} = ghg^{-1} : \quad h \in H < G$ this is a subgroup conjugate to H
If a supgroup $N < G$ has the property that any supgroup conjutates to N equals N (ie. $gNg^-1 = N \quad \forall g \in G$)
then N is a *normal subgroup*

$$N \lhd G$$

Exercice:
The kernel of any homomorphism

$$\varphi : G \Rightarrow H$$

is a *normal subgroup*

$$N \triangleleft G$$

$\mathbb{R}^n \triangleleft E_n$
$O_n < E_n$ is *not normal*
$E_n = \mathbb{R}^n \rtimes O_n$
$\tau_p O_n \tau \cdot p = (E_n)^p = \{g \in E_n : \quad g(p) = p\}$
if $q = g \cdot p$ for some action G on X
then $gG^p g^{-1} = G^q$

$$N \rtimes H \qquad \text{semiproduct}$$

is defined by an action if H on H by automorphism.
The euclidean motion
$x \Rightarrow v + Mx$ is

$$\left\{ \begin{array}{c} \text{orientation preserving if } \det M = +1 \\ \text{orientation reversing if } \det M = -1 \end{array} \right\}$$

$$SO_n := M \in O_n : \det(M) = \pm 1$$

$$SE_n := \{\text{orientaion preserving eucledian motion}\} = \mathbb{R}^n \rtimes SO_n$$

If -1 denotes any reflection in O_n
then $SO_n \rtimes \pm 1 = O_n$
(if n is odd, the antiposal map $-I$ is a orientation reversing element comutes with all rotations, so $O_n = SO_n \times \pm I$)